SMALL BODY POSE AND SHAPE ESTIMATION FROM SILHOUETTES VIA EXTENDED TARGET TRACKING
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Abstract. This work describes a method to estimate shape and pose of a small celestial body with optical measurements, and using only data extracted from the silhouette of the body, together with estimates of the position of the spacecraft from the ground navigation. Silhouettes are preferred to visual landmarks as the latter strongly depend on illumination angles, which vary during the mission. The shape of the small body is represented with a Gaussian Process, which naturally provides a continuous representation of the distance of the surface from the origin of the body and corresponding uncertainty. The proposed method shows promising results, offering an RMSE of approximately 1 % on the shape of 433 Eros with an initial prior for the shape of a sphere and 1-sigma uncertainties of 6 km, 250 m, 5 degrees, 5 degrees, and 1.7 degrees per hour, on the asteroid radius, asteroid position (in each direction), spin axis declination, ascension, and spin rate respectively.

Introduction. It is desired to enable missions to small bodies for which prior information about shape and spin is not precisely known. To this end, we present a recursive way to simultaneously estimate the visual hull and the pose of a small body. The visual hull is the largest shape that can be contained within the intersection between the cones generated by connecting every silhouette to their corresponding camera position, for every point in space.1 In 2 dimensions, the visual hull is equivalent to the convex hull, but in 3 dimensions it is much tighter than that. For the case of most celestial bodies, the difference between the visual hull and the actual shape is limited to the inside of craters and canyons. In previous work, we showed that extended target tracking is suitable when using lidar measurements.2 With this work, we show that, within the extended target tracking framework, using silhouette (or limb) information is equivalent to using lidar, at the cost of increased nonlinearities.

Methodology. We treat the asteroid as a target and leverage the extended target tracking framework to infer the state from multiple data points per time-step. An extended target is an object that may generate multiple measurements from the same sensor;3 via extended target tracking, it is usually possible to estimate the shape of the target and its orientation, together with its position. In this framework, Wahlstrom et al. used Gaussian processes (GPs) to estimate the shape, position, and orientation of a 2-dimensional star-convex target.4 A shape is star-convex if every point on its surface can be connected to a point inside of it by a straight line that never crosses the surface. While this description may sound limiting, said point can be arbitrary, and thus most shapes can be star-convex. An example of a shape that may be or not star-convex, depending on what point is chosen as its origin, is provided in Fig. 1. A GP is a stochastic process where each point is correlated to every other, with a correlation that decreases the farther the two points are; a GP f(x) can be seen as an infinite-dimensional Gaussian distribution and is fully described by its mean m(x) and covariance k(x, x′):5

\[ m(x) = \mathbb{E}[f(x)] \tag{1} \]
\[ k(x, x') = \mathbb{E}[(f(x) - m(x))(f(x') - m(x'))] \tag{2} \]

and the GP is:

\[ f(x) \sim \mathcal{GP}(m(x), k(x, x')). \tag{3} \]

The random process f(x) is what we aim to estimate: in this case, the distance of the surface from the center, in a certain direction x. The mean and covariance of a GP is updated for a finite number of basis points; from these points, it is possible to obtain an estimate with mean and standard deviation of the radius of the object anywhere on its surface. Because of this, the residual of a measurement can be computed anywhere, and the uncertainty due to the instrument is augmented to account for the uncertainty in the shape where the measurement occurred.

Exploiting the silhouette. An expected measurement of the silhouette is generated from the prior, and the point on the surface (together with the corresponding radial direction) that is expected to generate that measurement is saved. Then, the radius of the small body, in the previously saved direction, that would be required to generate the actual measurement is computed, and the residual is the difference between said computation and the value of

![Figure 1. An example of a shape that may be star-convex or not, depending on what point is chosen as the origin. Image Credit NASA/JPL.](image-url)
the radius in the prior. Hence, an angle measurement is transformed into a range measurement. Because of errors in the prior, the direction that is expected to generate the prior is not the one that generated the measurement; however, since in a GP every point is strongly correlated to all the points close to it, this error in the data association ends up being equivalent to a measurement error. Since said error is larger the farther away the prior is from convergence, nonlinear measurement underweighting is used.

**Results.** We do not generate pictures but instead directly obtain the measured silhouettes, which are are generated as follows: ray-tracing is done with a resolution of 0.2 degrees; then, only the points at the edge of the shape are kept. Again by ray-tracing, it is checked whether the points on the silhouette are shadowed from the sun by other parts of the body; those that are in shadows are removed. The data provided to the filter then consists of the angles of the remaining points. In future work we plan to use image generators together with silhouette extraction methods.

We apply this method to the estimation of pose and shape of Eros, an asteroid with a rather irregular shape (the distance between its surface and the center of mass ranges between 2.6 and 17.6 km). Eros has a revolution period of approximately 7 hours and we simulate a polar orbit with an approximately 20 hour period (initial semi-major axis of 35 km). The sun has an angle of approximately 15° with respect to the Equatorial plane. In our problem, we save the values of the GP for 1332 basis points, corresponding to angles equally spaced over the surface of a sphere.

The simulation time is approximately 7.5 days. One measurement every 35 minutes is taken. Measurement noise comes from errors in the prior (obtained from the ground) on the spacecraft position (1 σ error of 100 m in each direction) and pointing direction (1 σ error of 0.2 deg). 1 σ initial uncertainties are of 6 km, 250 m, 5 degrees, 5 degrees, and 1.7 degrees per hour, on the asteroid radius, asteroid position (in each direction), spin axis declination, ascension, and spin rate respectively. The initial shape prior is a sphere.

Figure 2 shows the RMS of the error obtained after 50 Monte Carlo runs. On average, the RMSE is approximately 300 m. However, a few regions have much larger errors. Two peaks (one at 5° S, 160° W, and another at 10° S, 15° E) are caused by the fact that the gradient of the radius is very large in those points, and the GP has a hard time following such rapid changes; however, in those two points the radius is almost parallel to the surface, and thus the error normal to the surface is much smaller than the displayed one. A larger region with a large error lies around 20° N and 100° W. It is likely that this error is caused by the mismatch between the actual shape and the corresponding visual hull. Finally, the whole southern region has large errors, caused by the shadow of the sun. When not including the shadows in the simulation, said region has errors of similar magnitude to the rest of the asteroid. For the parts of the asteroid that are visible, the average error is approximately 100 m. Figure 3 shows the convergence in the spin rate estimation. Aside from an initial transient, where there are a few outliers beyond the 3 σ lines, the filter behaves consistently and is unbiased.

**Conclusion.** We provide a method that makes use of silhouettes to estimate shape and pose of a small body, while requiring ground-based navigation. Silhouettes are preferred to visual landmarks as the latter strongly depend on illumination angles, which vary during the mission. We apply the method to the observation of 433 Eros, which has a radius ranging between 2.6 and 17.5 km. The obtained result is a map that has an average RMSE of 100 m for the visible regions. Since this method makes use of an extended Kalman filter, we expect that it should be possible to extend this method to the estimation of the S/C positions as well.

![Figure 2. RMSE in the shape estimation.](image2)

![Figure 3. Spin rate estimation.](image3)

**References.**


